Using The Second Digit Spreadsheets
by CHOUETTE

Introduction

Determination of the second digit of a root can be a big help in solving square and cube root cryptarithms. Unfortunately, the algorithm for calculating the second digit and the resulting subtrahend of a square root is complicated and time-consuming to use – and the algorithm for cube roots is even worse. The Second Digit spreadsheets (available as downloads on the ACA website) eliminate the drudgery by doing the calculations for you and speed the solution of such cryptarithms.

There are separate spreadsheets for square and cube roots, and the procedure for using them is the same. First, enter the base for the cryptarithm. The spreadsheet will then calculate all of the possible subtrahends for the chosen base and can be used for square and cube roots up to base 16.

Then, determine the possible first digit(s) of the root (indicated on the spreadsheet in red). Finally, scan down the column(s) headed by the possible first digit(s) to identify the possible subtrahend(s) and read the possible second digit(s) (indicated on the spreadsheet in blue) at the head of the row(s). The goal is to match the pattern(s) of the subtrahend(s) in the cryptogram to the pattern(s) of the subtrahend(s) on the spreadsheet.

Occasionally, you'll wind up with 3 or more possible second digits and more than a dozen candidate subtrahends. Fortunately, most of the possible subtrahends can usually be eliminated because of

conflicts and the correct second digit and subtrahend identified.

Base 10 Square Roots

Although square roots are among the easiest cryptarithms, the second digit method can speed the solution.

Consider ARIES' C-2 in the MA2023 *Cm*: no word, 9-0.

```
  O  A  T
√MU FF IN
 MA
  C FF
  A MA
    ST IN
    SC ON
     A AI
```

Obviously, I=0, C=A+1, and U=2A+1.
On the Square Root spreadsheet, enter 10 as the base. The possible values for O, MA, A, and AMA are:

O	MA	A	AMA	
4	16	6	616	x no such subtrahend
7	49	9	949	x no such subtrahend
8	64	4	464	x no such subtrahend
9	81	1	181	

Scanning down the 4 column, we see that 4 is impossible as the square root's second digit because the subtrahend 616 doesn't appear. Similarly, 7 and 8 are impossible as the square root's second digit. Thus, the second digit of the square root is 9 and, after entering the recovered letters into the key (indicated in capitals) the remaining letters (indicated in lower case) are easily determined.

```
9 8 7 6 5 4 3 2 1 0
O M s n f t U C A I
```

Base 10 Cube Roots

The procedure for using the second digit method for cube roots is the same as for square roots. Consider BION's C-3 in the MA2023 *Cm*: three words, 0-9.

```
  T   N   A
√TEN NES SEE
 UAT
 AHT NES
 AHA HTA
   U OEN SEE
   C UUE HON
   H NUP NUO
```

On the Cube Root spreadsheet, enter 10 as the base. The possible values of T, UAT, N, and AHAHTA are:

T	UAT	N	AHAHTA	
5	125	–	2*2**2	x no such subtrahend
6	216	–	1*1**1	x no such subtrahend
9	729	8	212192	

Scanning down the 5 column, we see that 5 is impossible as the cube root's second digit because there are no six digit subtrahends. Similarly, 6 is impossible as the cube root's second digit. Thus, the second digit of the cube root is 9 and, after entering the recovered letters into the key (indicated in capitals) the remaining letters (indicated in lower case) are easily determined.

```
0 1 2 3 4 5 6 7 8 9
s H A p e c o U N T
```

Roots in Other Bases

The spreadsheets are particularly helpful when dealing with roots in other bases. Consider BION's duodecimal cube root C-14 in the ND2011 *Cm*: three words, 0-B.

```
  O   E   V
√CH ISE LED
 VO
 OI ISE
 OA CUE
  U CCS LED
  U AVU SLV
    OOC LOT
```

On the Cube Root spreadsheet, enter 12 as the base. The possible values of O, VO, E, and OACUE are:

```
O VO E OACUE
3 23 - 3---- x no such subtrahend
4 54 A 40AB4 x O=E=4
4 54 B 46A2B
5 A5 9 55139 x O=A=5
```

Clearly, the second digit of the cube root is B. After entering the recovered letters into the key (indicated in capitals) the remaining letters (indicated in lower case) are easily determined.

```
0 1 2 3 4 5 6 7 8 9 A B
s h U t O V A l d i C E
```

An Alternate Method

The majority of the time you'll use the spreadsheets for pattern matching but, occasionally, you'll run across problems for which that's impossible. In that case, you'll use the spreadsheets to eliminate candidate subtrahends due to conflicts. Consider BION's duodecimal cube root C-13 in the JF2011 *Cm*: three words, 0-B.

```
  S   P   V
√PA RCH ING
 CS
 LH RCH
 LI OHP
  S HAA ING
  S IOA NGV
    CRN RAL
```

On the cube root spreadsheet, enter 12 as the base. There are three possible first digits – 3, 4, and 5. There are 5 subtrahend candidates in the 3 column, 7 in the 4 column, and 7 in the 5 column for a total of 19. At first, this appears like a lot of work but most of the candidate subtrahends can be eliminated because of conflicts. For example, all but one of the candidate subtrahends in the 3 column can be eliminated because of conflicts.

```
S CS  P LIOHP
3 23  5 10A75
3 23  7 17017 x I=P=7
3 23  8 1A368 x S=O=3
3 23  9 21899 x H=P=9
3 23  B 290BB x H=P=B
```

After scanning all three columns and eliminating conflicts, it turns out that only four possible second digits remain:

S	CS	P	LIOHP	0	1	2	3	4	5	6	7	8	9	A	B	
3	23	5	10A75	I	L	C	S		P		H			O		x
4	54	3	10923	I	L	H	P	S	C				O			x
4	54	8	31768		I		L	S	C	H	O	P				
4	54	9	37209	H		O	L	S	C		I		P			x

Clearly, the second digit of the cube root is 8. After entering the recovered letters into the key (indicated in capitals) the remaining letters (indicated in lower case) are easily determined.

0	1	2	3	4	5	6	7	8	9	A	B
g	I	r	L	S	C	H	O	P	v	a	n

Some additional examples

Base 10 Square Roots

JF2011 C-1. Square root (Two words, 1-0) LATIN DUDE

```
  T  T  D     A B D E M N O R S T
√ST RE ET
 NM          1 2 3 4 5 6 7 8 9 0
 BM RE
 BD MM
    NO ET
    OA TE
    BD RE
```

SO2008 C-1 Square root (Two words, 9-0) VERMONSTER

```
 N  B  S      A B D I K L N O R S
√L AB OR
 N           9 8 7 6 5 4 3 2 1 0
 I AB
 I RN
   NO OR
   ND LR
    L DK
```

Undecimal Square Roots

SO97 C-5 Undecimal square root (Two words, 0-A) FIBBER

```
  N  U  E  D      D E I J L N O R U W Y
√UN DE RL IE
 OE ______       0 1 2 3 4 5 6 7 8 9 A
  Y DE
  E WD _____
  I DU RL
  I DW OJ ___
     U IE IE
     J RR YE
     I OI LW
```

ND2003 C-12 Undecimal square root (Three "words", 0-A) E. TAMBIEN

```
 N  H  C         A C D E H I M N R T W
√T IH DM
 E _____         0 1 2 3 4 5 6 7 8 9 A
 C IH
 C EM ___
   AR DM
   AW EM
    E TD
```

Duodecimal Square Roots

MA2008 C-11 Duodecimal square root (Two words, 1-0) ARIES

```
  L  N  H          A E G H I L M N P R S W
√AE RI EL
 WL              1 2 3 4 5 6 7 8 9 A B 0
 SW RI
 SM RS
  S GL EL
    NH PS
    SP SA
```

ND98 C-8 Duodecimal square root (Two words, 0-1) FIBBER

```
  Y  T  M  Y       A B C E I L M P R T U Y
√CL IM AT IC
 CT              0 B A 9 8 7 6 5 4 3 2 1
  I IM
  C CT
  E EC AT
  E BI IM
     A TP IC
     A RT CT
       RU EE
```

Base 13 Square Roots

JF2001 C-12. Base thirteen square root. (Two words, 1-0) ARIES

```
  K  M  O        A B E G I K L M N O R U V
√MA RB LE
 EA_____        1 2 3 4 5 6 7 8 9 A B C 0
 TG RB
  N MN__
    LL LE
   UI AA___
    TE AI
```

JA 2001 C-11. Base 13 square root. (Two words, 0-C) ARIES

```
  M  A  N        A B C E G I K L M N P S T
√PA IN TS
 KS_____        0 1 2 3 4 5 6 7 8 9 A B C
  C IN
  A IK__
  L BI TS
    SG ME___
     S CC
```

Base 14 Square Roots

SO97 C-Sp-1. Base 14 square root. (Four words, D-0)
ARIES

```
 J  I  J  R      D E G H I J L N O P R S U X
√D IH LN XO
 H               D C B A 9 8 7 6 5 4 3 2 1 0
 U IH
 U JD
   GE LN
   GX UH
    X IH XO
    X JP XG
       GR SX
```

JF98 C-Sp-1. Base 14 square root. (Four words, 0-D)
ARIES

```
 O  Y  Y  L      A C D E I K L N O R S T W Y
√T YE DK WI
 D               0 1 2 3 4 5 6 7 8 9 A B C D
 D YE
 D TC
   WT DK
   WN AC
    C DS WI
    C LY TE
        S RS
```

Base 15 Square Roots

JF2002 C-Sp-1. Base 15 square root. (Three words, 0-1)
ARIES

```
  Y  S  U  A      A B G H I L N O P R S T U W Y
√WR ON GW AY
 AA             0 E D C B A 9 8 7 6 5 4 3 2 1
 TL ON
 TN LP
  T HR GW
  T AT RT
    TT OH AY
    TB IW RN
       HU IN
```

SO2015 C-Sp-2. Base 15 square root. (Two words, 0-1)
ARIES

```
 N  E  O  E      A C E F G H I L N O R S T W Y
√F RA GI LE
 C              0 E D C B A 9 8 7 6 5 4 3 2 1
 O RA
 O FS
   SL GI
   SS AY
    C CC LE
    C WA HS
      OT AG
```

Cube Roots

Base 8 Cube Root

MJ2015 C-8 Base 8 cube root. (One word, 0-7) HANNAH II

```
   S   U          A D H N O S T U
√HOU NDS          0 1 2 3 4 5 6 7
 HTT
  OU NDS
  OU ADU
     HTH
```

Base 10 Cube Roots

ND2017 C-2 Cube Root (Two words, 1-0) LI'L GAMIN

```
   R   O   L      A E F K L N O R S W
√SNO WFL AKE
 NAR____          1 2 3 4 5 6 7 8 9 0
 AAN WFL
  LO WSN___
  AW KWF AKE
  AN REK FRL
   A SLR SSR
```

JA2014 C-3 Cube Root (Four words, 0-1) THE DOC

```
  M   A   A       A B E H I M O S T W
√SO MEW HAT
 BM____           0 9 8 7 6 5 4 3 2 1
 OE MEW
 OI IHE___
  O TBI HAT
  O BEI EAE
    WST AHW
```

Undecimal Cube Roots

JF95 C-6 Undecimal cube root (No word, 0-A) MORDASHKA

```
   I   T          A D G I M N O P R S T
√RAN DTM
 ADS____          0 1 2 3 4 5 6 7 8 9 A
 ODM DTM
 OOP OID
  OM PGO
```

ND2006 C-11 Undecimal cube root (Three words, 0-A) BION

```
   C   N   P      A C D E F H I L N P S
√LAN DSC APE
 FIE____          0 1 2 3 4 5 6 7 8 9 A
 HDF DSC
 HEC HFN___
   D LEA APE
   L NHP DCL
   H ELE HEH
```

Duodecimal Cube Roots

JA91 C-9 Duodecimal cube root (Four words, 0-B) PIT

```
 A   S   S   R        A E I L M O P R S T U Y
√P ATR IOT ISM
 Y                    0 1 2 3 4 5 6 7 8 9 A B
 S ATR
 S IES
   SOP IOT
   SSI PLS
    RY LYU ISM
    PR LAI MLP
     R IEL UPR
```

MA2006 C-10 Duodecimal cube root (Three words, 0-B) BION

```
  O   C   O       A B C E H I L M N O U W
√HE NCH MEN
 UO               0 1 2 3 4 5 6 7 8 9 A B
 NU NCH
 NT ICW
  N IWH MEN
  I YOC WUO
    OYI MTL
```

Base 15 Cube Root

ND2007 BC-1 Base 15 cube root. (Two words, 0-E) GGMA

```
  C   M   N       A B C D E H I K L M N P R S Y
√AC DKB EKL
  M               0 1 2 3 4 5 6 7 8 9 A B C D E
  S DKB
  M HNC
  A CAM EKL
  A CAP BCR
    R LIY
```

CHALLENGE

SO2013 C-Sp-1. Double-key base 10 cube root. (TWO WORDS, 0-9, three words, 0-9) FORMALHAUT

```
 h   t   i   u    A E L M N O R S T V
√V ERM ONS TER
 M                0 1 2 3 4 5 6 7 8 9
 O ERM
 N ENL            a e h i l q t u w y
   tly auw
   twe ety        0 1 2 3 4 5 6 7 8 9
    OL ANE TER
    ui tuq ayu
    MS RSR ENE
```

MA2013 C-Sp-1. Double-key base 14 cube root. (FOUR WORDS, D-0, three words, 0-D) GGMA

```
  g   l   e   o      A B D F G H I M O P R S T U
√DRU GRA OTS DAR
 ytw                 D C B A 9 8 7 6 5 4 3 2 1 0
 IUH GRA
 tki gtt             d e g i k l n o r s t u w y
  IS MDT OTS
  td ksu woe         0 1 2 3 4 5 6 7 8 9 A B C D
   I GUH PBT DAR
     urd drs uno
     ITB DFS OFG
```